CENTRAL ASIA BUSINESS JOURNAL
WINTER 2021

CALL FOR PAPERS

The *Central Asia Business Journal*, published by KIMEP University quarterly, promotes understanding of business issues (broadly defined) in the region. As we see it, the region includes the post-Soviet "stans" (Kazakhstan, Kyrgyzstan, Tajikistan, Uzbekistan and Turkmenistan) as well as the post-Soviet states of the trans-Caucasus area (including Armenia, Azerbaijan and Georgia). The ISSN number is 1991-0002. The *Journal* is registered with the information ministry in Kazakhstan.

Central Asia is a fertile area for research. It prospers from rich natural resources and high commodity prices as well as from its location at the crossroads of East and West. But its open economy is vulnerable to such external shocks as the global financial crisis of 2008, and its Soviet legacy complicates its transition to markets.

Authors may submit research papers, case studies, and book reviews. We also invite students' papers. All submissions must be in English. All submissions are peer-reviewed, usually on a double-blind basis. The deadline for submissions to the Spring 2022 issue is February 1; we will consider later submissions for later issues.

The journal is open to all methodologies, but it especially welcomes papers that are conceptually and analytically strong and that relate to the real world. We prefer papers with new findings but also publish surveys. All papers should discuss applications to Central Asia.

The journal's interests include:

Behavioral economics
Business cycles and economic development
Business law
Corporate governance
Emerging markets
Financial and capital markets and industries
Human resources management
Institutional economics
International accounting standards and taxation
International business and globalization
Leadership
Logistics and supply chain management
Management information systems
Marketing strategies and effectiveness
Market integration and segmentation
Market structure and efficiency
Mathematical economics
Microfinance and development
Multinational enterprises and business strategy
Natural resources and their internationalization
Nongovernmental organizations and entrepreneurs
Political economy
Risk and uncertainty
Statistical economics
Tourism and the hospitality business

We also welcome contributions to three sections of the journal:
Perspectives. This features nontechnical surveys of issues in Central Asian business that would interest scholars and policymakers. An example is a survey of theoretical and empirical papers about customs unions. A typical length is 4,000 to 6,000 words. Please propose your topic to the managing editor before beginning work.
Book reviews. Reviews should summarize and evaluate books about Central Asian business or about business issues that interest the region. Most reviews will concern recent books, but the journal may also publish a retrospective essay about well- known titles in a given field. A typical length for a review is 1,500 to 2,500 words. Please write the managing editor about the book that you propose to review.
Symposium. This consists of several commentaries on a recent issue of interest – for example, the August 2015 float of the tenge. A typical commentary may run 1,500 to 2,500 words. Usually, the journal commissions commentaries, but you may propose a symposium to the managing editor.
We try to give the author a decision on her submission in six weeks.
The *Journal*'s website, www.kimep.kz/bang-college-of-business/central-asia-business-journal, provides guidelines for authors and recent issues.
The *Journal* is published by KIMEP University, 4 Abai Prospekt, Almaty, Kazakhstan 050010. For further information and submissions, please write to the Journal's managing editor, Leon Taylor, at ltaylor@kimep.kz.
We thank Irina Kovaleva and Elmira Mukanova for efficient staff support and translation.
The title page of this journal is based on a Microsoft Word template.

Table of Contents (Volume 12, Issue 4)

Characteristics of Innovative Firms in Kazakhstan

Alma Alpeissova[1]
KIMEP University, Kazakhstan
alpeis@kimep.kz
and
Mira Nurmakhanova
KIMEP University, Kazakhstan
miranur@kimep.kz

Abstract: Although the literature delineates factors affecting innovation in transitional economies, few studies explore characteristics of innovation at the firm level. This study investigates characteristics of firms such as size, age, the form of ownership (state or private, local or foreign), and gender of the owner. This study applies ANOVA and t-tests to data from the 2019 World Bank Enterprise Survey (ES).

JEL classifications: O31, O32, M1

Keywords: Innovation Performance, Innovation Management, Determinants of Innovative Output

1. Introduction

The business environment drives nations and companies to find new ways to strengthen their competitive positions and sustain growth. The innovative performance of contemporary firms determines their financial success (Bloom, Jones, Van Reenen, & Webb, 2020). Recently the Kazakhstani government has established mechanisms, institutions, and infrastructure to support businesses in innovating.

However, in addition to government support, the ability of the company to innovate depends on many factors. These are not limited to the expertise of the company to invent; they include experience, managerial competence, financial and other resources as well as the firm's structure and the "whole innovation process management that is supposed to be fundamentally and systematically inbuilt in the company's operations" (Tidd & Bessant, 2020).

The resource-based view (RBV) states that any company needs to accumulate knowledge to be able to innovate (Lukovszki, Rideg, & Sipos, 2021). The RBV considers organizational resources (tangible and intangible) as inputs offering strategic options to firms (Wernerfelt, 1984). Furthermore, the "recombination of resources, activities, and linking routines within the firm (i.e.

[1] The corresponding author.

firm-level factors) lead to innovative forms of competitive advantage" (Mathews, 2006; Kostopoulos, Spanos, & Prastacos, 2002). Innovation is the organization's ability "to develop new or improved products or services and its success in bringing those products or services to the market" (Gumusoglu & Ilsev, 2009).

There is a lack of Kazakhstani research on firm-level innovation within the local business context that can provide recommendations to government policymakers and business decision-makers. An exception is recent investigation of the effect of political decisions and macroeconomic factors on innovation development (Satpayeva, 2017; Korgan, Sabirova, & Adietova, 2019). This paper fills the gap by reporting on relations found between the firm's characteristics (such as size, age, and owners' origin and gender) and its innovative performance.

This paper consists of five sections. The discussion of literature precedes that of methodology. The third section discusses the major findings about the characteristics of innovative firms. The last section conveys conclusions, study limitations, and recommendations for further research.

2. Related Literature

For two decades, research on characteristics of innovative firms has been popular in the field of management.

2.1 Firm Size

The literature on the relation between a firm's size and its innovative performance does not provide clear evidence on whether it is positive. Some studies show significant positive correlations (Patel & Pavitt, 1992; Damanpour, 1992; Cohen & Klepper, 1996; Camison-Zornoza, Lapiedra-Alcami, Segarra-Cipres, & Boronat-Navarro, 2004; Noori, Narasbadi, Yazdi, & Babakhan, 2017). But several studies show either no relationship (Laforet & Tann, 2006; Marsili & Salter, 2006; Baregheh & Nemsworth, 2016) or a negative relationship (Salavou, Baltas, & Lioukas, 2004; Shefer & Frenkel, 2005). These variations are explained by differences in the measures of innovative performance and firm size as well as by the impact of other factors such as industry and location. As innovation requires significant investment (Winters & Stam, 2007), a firm's investment in research and development (R&D) may be proportionate to its size. Accordingly, this study proposes the following hypothesis:
H1: There is a significantly positive relationship between firm size and innovative activity.

2.2 Firm Age

The experience that the firm accumulates over time affects positively its ability to innovate (Sørensen & Stuart, 2000; Winters & Stam, 2007). However, there is also evidence that firm age negatively affects innovation quality (Balasubramanian & Lee, 2008). Older firms have more opportunities for innovation due to the learning effect (Cohen & Levinthal, 1989, 1990), developed networks, and established processes and routines (Bierly & Daly, 2007). On the other hand, entrant firms have a higher probability of innovation in comparison with the oldest firms (Huergo & Jaumandreu, 2004). This phenomenon may be explained by the fact that older firms focus on non-radical options and rely on knowledge (Klette & Kortum, 2004; Akcigit & Kerr, 2018). Given the capacity to innovate, the older firm innovates more than the younger one; but the younger firm innovates more when neither firm has developed the ability to innovate (Withers, Drnevich, & Marino, 2011).
This study proposes the following hypothesis:
H2: There is a significantly positive relationship between firm age and innovative activity.

2.3 Firm's Form of Ownership: State or Private, Local or Foreign

The literature suggests that a firm's decision to innovate depends highly on ownership structure (Lee & O'Neill, 2003; Lee, 2005) due to the risky, uncertain, and long-term investments involved.
Recent evidence of a positive relation between the growth of private firms and high-intensity R&D could be found in innovation-related studies in China (Jefferson, Hu, Guan, & Yu, 2003). Although the state firm, unlike private firms, has more R&D resources, their subsequent use is less efficient than in private firms (Zhou, Gao, & Zhao, 2017; Yang et al., 2020). But government support is especially important in emerging economies for building innovative capability (Wang, Jin, & Banister, 2019). Several authors support that private ownership leads to innovative activities (Zahra, Ireland, & Hitt, 2000), but some researchers believe that conflicts of interest between investors and executives may prevent firms from investing in R&D (Holmstrom, 1989). Subsequent studies revealed a positive relation between R&D spending and institutional ownership rather than individual ownership (Baysinger, Kosnik, & Turk, 1991; Hansen & Hill, 1991) as institutional investors have more resources and information.
Foreign ownership relates positively to innovation (Love, Ashcroft, & Dunlop, 1996; Choi, Park, & Hong, 2012) due to the resources, technologies, and other firm-specific assets and capabilities of multinational enterprises (MNEs) (Dachs, Ebersberger, & Pyka, 2008). For instance, foreign companies have greater access to foreign research and development centers (Dzikowski & Tomaszewski, 2014), and invest heavily in machinery and equipment (Goedhuys, 2007) as well as in human capital. For example, foreign firms train intensively on operating new vintage machinery and equipment (Blomström & Kokko, 2003). In addition, "the most important mechanism of technology transfer for developing capabilities in locally-owned firms [is] the knowledge acquired by local personnel when they worked for foreign subsidiaries" (Iammarino, Padilla, & von Tunzelmann, 2008). Building innovative capabilities depends on foreign expertise that is transferred to the economy through MNEs.
This study proposes the following hypotheses:
H3: Private companies have a significant positive relation to innovation.
H4: Foreign companies have a significant positive relation to innovation.

2.4 Gender and Innovative Performance

The concept of gender is new in innovation literature and therefore is understudied (Fagerberg, Mowery, & Nelson, 2005). Several studies indicated that innovations among female employees are rare, due to unfair organizational culture and practices (Cropley & Cropley, 2017). However, Nählinder, Tillmar, and Wigren (2015) found previous studies on gender and innovation did not control for traditionally male-dominated industries, therefore containing a bias towards female innovation. This study found no relation between gender and innovation.
Several studies state that compared to female-owned firms, more innovative activities are associated with male-owned companies (Marvel, Lee, & Wolfe, 2015; Chen, Chen, Hsu, & Podolski, 2016; Alves, Galina, Macini, Carvalho, & Costa, 2017), because women are more averse to risk and competition than men (Croson & Gneezy,2009).
This study proposes the following hypothesis:
H5: There is a significantly positive relationship between the gender of the firm's owner (specifically, firms having female owners) and innovation.

3. Methodology

This study is based on data from the 2019 World Bank Enterprise Survey,conducted from January to

October 2019 in Kazakhstan, jointly by the European Bank for Reconstruction and Development, the European Investment Bank, and the World Bank Group. The unit in the study was the establishment, the physical location of the business. The establishment must make its own financial decisions.

The sample was selected randomly, with stratification across 11 regions of oblasts and major cities. The sample consisted of 1446 establishments representing 921 firms from manufacturing, 174 retail, and 151 other services.

Table 1

Variables, Definitions, and Descriptive Statistics

Variable	**Definition**	**Descriptive statistics**		
		Mean	**Min**	**Max**
Innovative activity	The firm introduced new or significantly improved products or services in the past three years	1.72	1 (yes)	2 (no)
Firm size	Number of permanent employees in the company	73.34	1	3500
Firm age	Year establishment began operations	2003.87	1927	2017
Form of ownership	Share of the sample firms with state ownership	4.37	1 (state/yes)	2 (state/no)
Ownership origin	Share of the sample firms with foreign ownership	0.57	1 (foreign/yes)	2 (foreign/no)
Gender of the owner	Share of the sample firms with females among the owners	1.676	1 (female/yes)	2 (female/no)

The descriptive statistics in Table 1 indicate that 28% of the sample firms had introduced innovation in the prior three years. The average number of permanent employees working in the sample firms was 73 and the largest firm that participated in the study employed 3,500. The youngest firm was established in 2017, and the oldest firm had 92 years of operations. The share of the sample firms with state ownership was 4.37%. The firms with foreign ownership represented 0.57% of the sample, and 1.676% of firms had females among owners.

To evaluate Hypotheses 1-5, the study calculated one-way ANOVA and the t values to test for statistically significant differences between the mean values of two groups of firms (innovative and not innovative) with respect to these characteristics: Firms owned by males or females, those owned by the state or by private owners, those with foreign ownership or purely local ownership, those that are new or those that are old, and those that are small or large.

4. Findings and Analysis

Tables 2 and 3 report the test results. Preliminary tests validated the assumptions of normality, homogeneity of variances, linearity, and reliability of data.

Table 2

Results of ANOVA and Proportion Tests

Variable	**Test**	**P-value**	**Mean (non-innovative)**	**Mean (innovative)**
Size	ANOVA	0.0005	64.24	96.72
Age	ANOVA	0.0005	2004.27	2002.82
State ownership	ANOVA	0.74	Not Significant	
Foreign ownership	ANOVA	0.0002	0.036	0.071
Gender	Proportion Test	0.285	Not Significant	

Table 3
Results of T-Test

Variable	**P-value**	**Mean (non-innovative)**	**Mean (innovative)**
Size	0.0007	64.75	96.79
Age	0.0003	2004.35	2002.83
State ownership	0.53	Not Significant	
Foreign ownership	0.0002	0.034	0.069
Gender	0.28	Not Significant	

Tables 2 and 3 indicate significant relationships of innovation to size (p-values equal to 0.0005 (ANOVA) and 0.0007 (t-test)), age (p-values equal to 0.0005 (ANOVA) and 0.0003 (t-test), and foreign ownership (p-values equal to 0.0002 (ANOVA) and 0.0002(t-test)). No significant relations were found between innovation and the firm's form of ownership (p-values equal to 0.74 (ANOVA) and 0.53 (t-test)), and gender of the owner (p-values equal to 0.285 (ANOVA) and 0.28 (t-test)).
There are differences in the means of firms' size, age, and foreign ownership for innovative and non-

innovative firms. For instance, the means for size are 64.24 (ANOVA) and 64.75 (t-test) for non-innovative firms, and 96.72 (ANOVA) and 96.79 (t-test) for innovative firms. Both tests indicate that larger firms show significantly positive relations with innovative performance. The means for firms' age also differ, i.e. 2002.82 (ANOVA) and 2002.83 (t-test) for innovative, and 2004.27 (ANOVA) and 2004.35 (t-test) for non-innovative firms. Older firms are more associated with innovation than young firms. The ANOVA test indicates no significant relations between innovation and state ownership or owner gender. Thus the tests supported Hypotheses 1, 2, and 4 and rejected Hypotheses 3 and 5.

5. Discussion

Innovation management is considered in terms of the mechanism or pre-condition that enables firms to sustain innovation by adapting to environmental changes and intense competition, and by bringing technology and new products to the market (Utterback, 1994; Dougherty & Hardy, 1996; Lam, 2005; Razavi & Attarnezhad, 2013).

The internal determinants of innovation shed light on the innovative potential of the entire region. In addition, they serve as a starting point for understanding how the state can support firm-level innovation.

This study reports statistically significant positive correlations between the firm's size or age and innovative performance. These results support the RBV theory that the company's resources (conceptualized as accumulated experience and expertise) improve innovation. Many researchers argue that large organizations have more complex and diverse resources (such as financial slack, marketing skills, research capabilities, and more qualified professionals) able to support more innovation (Nord & Tucker, 1987; Damanpour, 1992). These findings indicate a need for state support in the form of financial and technical resources for small and medium-sized enterprises (SMEs) in Kazakhstan, since SMEs drive innovation and economic performance in various regions.

Foreign companies innovate more than local companies. This result supports previous research that stresses the importance of MNEs as the channel to transfer knowledge and technologies (Hoekman & Javorcik, 2006). Firms use several channels to acquire new technologies, such as patents, licensing, internal and outsourced R&D, foreign partners, and foreign suppliers. (Hoekman & Jaorcik, 2006). Traditionally, MNEs have increased their control of their technological competencies and have hesitated to transfer knowledge and expertise to their subsidiaries; but there are recent modifications of their global strategy. To adapt more products and processes to host markets (Mansfield, Teece, & Romeo, 1979), MNEs have diversified technological competencies. Thus they absorb and integrate knowledge and capacities throughout their international networks, leading to more innovation in host countries (Iammarino & McCann, 2013). Moreover, MNEs have become keener to establish R&D alliances with foreign companies (Castellani & Zanfei, 2007). Many states support MNEs for their technology and knowledge transfer.

6. Conclusion

This study reveals positive and significant correlations between some firms' characteristics such as firm age, size, and foreign ownership with their innovative performance. These results contribute to the literature by adding empirical evidence that more experienced and larger firms have more resources and capabilities to innovate than other firms. Also, companies with foreign ownership have a better innovative performance than local companies. The policy implications of the literature regarding MNEs and joint ventures with foreign partners highlight their value in transferring and adopting technology (Hoekman, Maskus, & Saggi, 2005).

There is a need for more elaborate research.For instance, research should include more characteristics (e.g., top management expertise and experience, the company's absorptive capacity, qualifications of employees, and corruption). This paper is limited to industry- or location-specific comparisons. It is limited to identifying causality, magnitude, and sign of correlations.
In short, this study is a preliminary analysis of any relationships between the firm's characteristics and its innovative performance in Kazakhstan. The next step of the research wouldfocus on joint analysis of all of these characteristics in one model, with firms' innovative performance as dependent variables and their characteristics as independent ones.

Alma Alpeissova is an Assistant Professor of the Management and Marketing Department, KIMEP University. She earned her DBA at KIMEP University, Kazakhstan.

Mira Nurmakhanova is an Associate Professor of the Accounting and Finance Department, KIMEP University. She earned her Ph.D. at Iowa State University.

7. Summary

English: This study investigates the characteristics of innovative firms in Kazakhstan, drawing upon World Bank data for 1,446 firms. It uses ANOVA and t-tests to identify the impact of firm characteristics such as size, age, form of ownership, and gender of the owner. Size, age, and foreign origin have statistically significant impacts on innovation.

Russian: данное исследование направлено на изучение характеристик инновационных организаций, основываясь на данных Всемирного Банка, собранных у предприятий Казахстана в 2019 году. Статистический анализ ANOVA и T-тестов для 1446 компаний, участвующих в данном исследовании выявил, что такие характеристики компании, как ее размер и возраст оказывают статистически значимое влияние на ее инновационную деятельность. Также, иностранные компании в Казахстане больше вовлечены в инновационную деятельность, чем местные компании.

Kazakh: Бұл зерттеу Дүниежүзілік банктің 2019 жылы Қазақстандағы кәсіпорындардан жиналған деректеріне негізделген инновациялық ұйымдардың сипаттамаларын зерттеуге бағытталған. 1446 компания арасында жүргізілген ANOVA және T-тесттерінің статистикалық сараптамасы көрсеткендей, компанияның өмір сүру мерзімі мен көлемі оның инновациялық іс-әрекетіне әсер ететіндігі айқындалған. Сонымен қатар, Қазақстандағы шетелдік компаниялардың жергілікті компанияларға қарағанда инновациялық іс-әрекеті қарқынды.

8. References

Akcigit, U., & Kerr, W. R. (2018). Growth through heterogeneous innovations. *Journal of Political Economy*, *126(4)*, 1374-1443.

Alves, M. F. R., Galina, S. V. R., Macini, N., Carvalho, L. C., & Costa, T. (2017). Internationalization and innovation in nascent companies: Does gender matter? *Journal of Small Business and Enterprise Development*, *24(4),* 887-905.

Balasubramanian, N., & Lee, J. (2008). Firm age and innovation. *Industrial and Corporate Change*, *17(5)*, 1019-1047.

Baregheh, A., Rowley, J., & Hemsworth, D. (2016), The effect of organizational size and age on position and paradigm innovation. *Journal of Small Business and Enterprise Development*, *23(3),* 768-789.

Baysinger, B. D., Kosnik, R. D., & Turk, T. A. (1991). Effects of board and ownership structure on corporate R&D strategy. *Academy of Management Journal*, *34(1)*, 205-214.

Bierly III, P. E., & Daly, P. S. (2007). Alternative knowledge strategies, competitive environment, and organizational performance in small manufacturing firms. *Entrepreneurship Theory and Practice*, *31(4)*, 493-516.

Blomström, M., & Kokko, A. (2003). *Human capital and inward FDI* (Vol. 3762). London: Centre for Economic Policy Research.

Bloom, N., Jones, C.I., Van Reenen, J., & Webb, M. (2020), Are ideas getting harder to find?, *American Economic Review*, *110(4),* 1104-1144.

Camison-Zornoza, C., Lapiedra-Alcami, R., Segarra-Cipŕes, M., & Boronat-Navarro, M. (2004). A meta-analysis of innovation and organizational size. *Organization Studies*, *25*, 331-361.

Castellani, D., & Zanfei, A. (2007). Internationalisation, innovation, and productivity: How do firms differ in Italy? *World Economy*, *30(1)*, 156-176.

Chen, C., Chen, Y., Hsu, P. H., & Podolski, E. J. (2016). Be nice to your innovators: Employee treatment and corporate innovation performance. *Journal of Corporate Finance*, *39*, 78-98.

Choi, S. B., Park, B. I., & Hong, P. (2012). Does ownership structure matter for firm technological innovation performance? The case of Korean firms. *Corporate Governance: An International Review, 20,* 267–288.

Cohen, W. M., & Klepper, S. (1996). Firm size and the nature of innovation within industries: The case of process and product R&D. *The Review of Economics and Statistics*, *78 (2),* 232- 243.

Cohen, W. M., & Levinthal, D. A. (1989). Innovation and learning: The two faces of R & D. *The Economic Journal*, *99(397),* 569-596.

Cohen, W. M., & Levinthal, D. A. (1990). Absorptive capacity: A new perspective on learning and innovation. *Administrative Science Quarterly*, *35 (1)*, 128-152.

Cropley, D., & Cropley, A. (2017). Innovation capacity, organisational culture and gender. *European Journal of Innovation Management*, *20(3),* 493–510.

Croson, R., & Gneezy, U. (2009). Gender difference in preferences. *Journal of Economic Literature, 47(2),* 448–474.

Dachs B., Ebersberger B., Pyka A. (2008). Why do firms cooperate for innovation? A comparison of Austrian and Finnish CIS3 results. *International Journal of Foresight and Innovation Policy*, *4(3/4),* 200-229.

Damanpour, F. (1992). Organizational size and innovation. *Organization Studies*, *13(3),* 375–402.

Dougherty, D., & Hardy, C. (1996). Sustained product innovation in large, mature organizations: Overcoming innovation-to-organization problems. *Academy of Management Journal*, *39(5)*, 1120-1153.

Dzikowski, P., & Tomaszewski, M. (2014). The impact of firm size and its ownership on innovation cooperation in medium-high and high technology sectors in Poland. *Management*, *18(1),* 385-396.

Fagerberg, J., Mowery, D. C., & Nelson, R. R. (Eds.). (2005). *The Oxford handbook of innovation*. Oxford: Oxford University Press.

Goedhuys, M. (2007). Learning, product innovation, and firm heterogeneity in developing countries: Evidence from Tanzania. *Industrial and Corporate Change*, *16(2)*, 269-292.

Gumusoglu, L., & Ilsev, A. (2009). Transformational leadership creativity, and organizational innovation. *Journal of Business Research, 62*, 461–473.

Hansen, G. S., & Hill, C. W. (1991). Are institutional investors myopic? A time-series study of four technology-driven industries. *Strategic Management Journal*, *12(1),* 1-16.

Hoekman, B. M., & Javorcik, B. S. (Eds.). (2006). *Global integration and technology transfer*. Washington, D. C.: World Bank Publications.

Hoekman, B. M., Maskus, K. E., & Saggi, K. (2005). Transfer of technology to developing countries: Unilateral and multilateral policy options. *World Development*, *33(10),* 1587-1602.

Holmstrom, B. (1989). Agency costs and innovation. *Journal of Economic Behavior & Organization*, *12(3),* 305-327.

Huergo, E., & Jaumandreu, J. (2004). How does the probability of innovation change with firm age? *Small Business Economics*, *22(3)*, 193-207.

Iammarino, S., & McCann, P. (2013). *Multinationals and economic geography: Location, technology, and innovation*. Cheltenham, England: Edward Elgar Publishing.

Iammarino, S., Padilla, R., & von Tunzelmann, N. (2008). Technological capabilities and global-local interactions: The electronics industry in two Mexican regions. *World Development, 36*, 1980–2010.

Jefferson, G., Hu, A., Guan, X., & Yu, X. (2003). Ownership, performance, and innovation in China's large and medium-size industrial enterprise sector. *China Economic Review, 14(1)*, 89–113.

Klette, T. J., & Kortum, S. (2004). Innovating firms and aggregate innovation. *Journal of Political Economy*, *112(5)*, 986-1018.

Korgan, B.B., Sabirova, R.K., & Adietova, E.M. (2019). Innovative economy of Kazakhstan. *News of the National Academy of Sciences of the Republic of Kazakhstan Series of Social and Human Sciences*, *4(No. 326)*, 123-129.

Kostopoulos, K.C., Spanos, Y.E., & Prastacos, G.P. (2002). The resource-based view of the firm and innovation: Identification of critical linkages. The 2nd European Academy of Management Conference, EURAM, Stockholm.

Laforet, S., & Tann, J. (2006). Innovative characteristics of small manufacturing firms. *Journal of Small Business and Enterprise Development*, *13(3)*, 363-380.

Lam, A. (2005). Organizational innovation. In Fagerberg, J., Mowery, D. C., & Nelson, R. R. (Eds,), *The Oxford Handbook of Innovation*. Oxford: Oxford University Press, 115–47.

Lee, P.M, & O'Neill, H.M. (2003). Ownership structures and R&D investments of U.S. and Japanese firms: Agency and stewardship perspectives. *Academy of Management Journal*, *46*, 212-225.

Lee, P. (2005). A comparison of ownership structures and innovations of U.S. and Japanese firms. *Managerial and Decision Economics, 26*, 39-50.

Love, J. H., Ashcroft, B., & Dunlop, S. (1996). Corporate structure, ownership, and the likelihood of innovation. *Applied Economics*, *28(6)*, 737-746.

Lukovszki, L., Rideg, A. & Sipos, N. (2021). Resource-based view of innovation in SMEs: An empirical analysis based on the global competitiveness project. *Competitiveness Review: An International Business Journal*, *31(3),* 513-541.

Mansfield, E., Teece, D., & Romeo, A. (1979). Overseas research and development by US-based firms. *Economica*, *46(182),* 187-196.

Marsili, O., & Salter, A. (2006). The dark matter of innovation: Design and innovative performance in Dutch manufacturing. *Technology Analysis & Strategic Management*, *18(5)*, 515-534.

Marvel, M.R., Lee, I.H.I., & Wolfe, M.T. (2015). Entrepreneur gender and firm innovation activity: A multilevel perspective. *IEEE Transactions on Engineering Management, 62(4)*, 558–567.

Mathews, J.A. (2006). Resource and activities are two sides of the same coin: Duality of the activities and resource-based views of strategic management. The Conference on Strategic Management, Copenhagen.

Nählinder, J., Tillmar, M., & Wigren, C. (2015). Towards a gender-aware understanding of innovation: A three-dimensional route. *International Journal of Gender and Entrepreneurship, 7(1),* 66–86.

Noori, J., Nasrabadi, M. B., Yazdi, N., & Babakhan, A. R. (2017). Innovative performanceof Iranian knowledge-based firms: Large firms or SMEs? *Technological Forecasting and Social Change*, *122*, 179-185.

Nord, W. R., & Tucker, S. (1987). *Implementing routine and radical innovations.* Lexington, MA: Lexington Books.

Patel, P., & Pavitt, K. (1992). The innovative performance of the world's largest firms: Some new evidence. *Economics of Innovation and New Technology*, *2(2),* 91-102.

Razavi, S. H., & Attarnezhad, O. (2013). Management of organizational innovation. *International Journal of Business and Social Science*, *4(1),* 226-232.

Salavou, H., Baltas, G., & Lioukas, S. (2004). Organizational innovation in SMEs: The importance of strategic orientation and competitive structure. *European Journal of Marketing, 38(9/10),* 1091-1112.

Satpayeva, Z. T. (2017). State and prospects of development of Kazakhstan innovative infrastructure. *European Research Studies Journal, 20(2B)*, 123-148.

Shefer, D., & Frenkel, A. (2005). R&D, firm size, and innovation: An empirical analysis. *Technovation*, *25(1)*, 25-32.

Sørensen, J. B., & Stuart, T. E. (2000). Aging, obsolescence, and organizational innovation. *Administrative Science Quarterly*, *45(1)*, 81-112.

Tidd, J., & Bessant, J.R. (2020). *Managing innovation: Integrating technological, market, and organizational change*. Hoboken, NJ: John Wiley & Sons.

Utterback, J. M. (1994). Radical innovation and corporate regeneration. *Research Technology Management*, *37(4)*, 10.

Wang, L., Jin, J.L., & Banister, D. (2019). Resources, state ownership and innovation capability: Evidence from Chinese automakers. *Creativity and Innovation Management*, *28(2)*, 203–17.

Wernerfelt, B. (1984). A resource-based view of the firm. *Strategic M anagement Journal*, *5(2),* 171-180.

Winters, R., & Stam, E. (2007). Beyond the firm: Innovation and networks of high technology, SMEs. In Arauzo, J.M., & Majon, M. (Eds.), *Entrepreneurship, industrial location, and economic , growth.* Cheltenham, England: Edward Elgar, 235-252.

Withers, M. C., Drnevich, P. L., & Marino, L. (2011). Doing more with less: The disordinal implications of firm age for leveraging capabilities for innovation activity. *Journal of Small Business Management*, *49(4)*, 515-536.

Yang, H., Steensma, H. K., & Ren, T. (2020). State ownership, firm innovation, and the moderating role of private-sector competition: The case of China. *Competitiveness Review: An International Business Journal, 31(4),* 726-746. Retrieved from https://doi.org/10.1108/CR-02-2019-0024.

Zahra, S. A., Ireland, R. D., & Hitt, M. A. (2000). International expansion by new venture firms: International diversity, mode of market entry, technological learning, and performance. *Academy of Management Journal*, *43(5),* 925-950.

Zhou, K.Z., Gao, G.Y., & Zhao, H. (2017). State ownership and firm innovation in China: An integrated view of institutional and efficiency logics. *Administrative Science Quarterly*, *62(2)*, 375-404.

An Exact Polynomial-Time Algorithm for the Optimal Solution of Traveling Salesman Problems

Vassilly Voinov,
Independent scholar, Kazakhstan
voinovv@mail.ru
and
Natalya Pya Arnqvist,
Umeå University, Sweden
natalya.pya@umu.se

Abstract: *A polynomial-time algorithm for the optimal solution of traveling salesman problems (TSPs) is introduced. The algorithm is based on a method for constructing all 0-1 non-negative integer solutions of a linear Diophantine equation proposed by Voinov and Nikulin in 1997, and a simple sub-tours elimination technique. Computational computer experiments confirmed that this algorithm, contrary to heuristic approaches, constructs all existing optimal solutions for symmetrical and asymmetrical TSPs, and that its time-complexity is* $O(n^{<6})$. *Applications of the algorithm are discussed.*

AMS subject classifications (2010): *05A15, 68Q17, 90C10, 90C27, 90C57, 97M40*

Keywords: *Discrete optimization, traveling salesman problem, linear Diophantine equations, integer programming, sub-tours elimination*

1. Introduction

As a mathematical puzzle, the *Traveling Salesman Problem* (TSP) was first formulated in 1930. Since that time a huge amount of research has been conducted, but an efficient method to solve TSP has not been found. Even the well-known deterministic solutions to the problem possess an exponential time complexity. That is why the TSP is considered to be NP-hard (nondeterministic polynomial time). At the same time the problem possesses many important applications: In computer wiring, wallpaper cutting, holes punching, job sequences, the structure of crystals analysis, aircraft mission planning, material handling in a warehouse, clustering of data arrays, the orienteering problem, integrated chip testing, parcels collection and sending, DNA sequences, etc. Consult, e.g., reviews of Laporte (1992), Pataki (2003), Goyal (2010), and Sathya and Muthukumaravel (2015). Currently, in practice, people use exact algorithms based on integer linear programming formulations (ILP) or approximate heuristic algorithms.

Consider briefly the ILP formulation of Dantzig, Fulkerson and Johnson (DFJ) (1954) that is used in this research. Let $G = (V, A)$ be a graph with a set V of n vertices, and let A be a set of arcs or edges. Let C be an $n \times n$ distance or cost matrix associated with A with elements c_{ij} being positive integers, $i, j \in V,\ i \neq j$. The TSP means to find a minimum Hamiltonian circuit of length L that passes through each vertex once and only once. The DFJ formulation is

$$\text{minimize} \quad L = \sum_{i \neq j} c_{ij}\delta_{ij} \tag{1.1}$$
$$\text{subject to} \quad \sum_{j=1}^{n} \delta_{ij} = 1,\ i = 1, \dots, n, \tag{1.2}$$
$$\sum_{i=1}^{n} \delta_{ij} = 1,\ j = 1, \dots, n, \tag{1.3}$$
$$\sum_{i,j \in S} \delta_{ij} \leq |S| - 1,\ S \subset V,\ 2 \leq |S| \leq n - 2, \tag{1.4}$$
$$\delta_{ij} \in \{0,1\},\ i,j = 1, \dots, n,\ i \neq j. \tag{1.5}$$

Constraints (1.2) and (1.3) are degree constraints. Constraints (1.4) are sub-tour elimination constraints. An alternative equivalent form of constraints (1.4) is Laporte (1992)

$$\sum_{i \in S} \sum_{j \in V \setminus S} \delta_{ij} \geq 1, S \subset V, 2 \leq |S| \leq n - 2. \tag{1.6}$$

The last constraints are also known as connectivity constraints.

In the above DFG formulation there are n for symmetrical or $2n$ for asymmetrical TSP degree constraints and $2^n - 2n - 2$ sub-tour elimination constraints. Since the number of sub-tour elimination constraints increases exponentially, Laporte (1992, p. 234) concluded that "even for moderate values of n, it is unrealistic to solve DFJ directly by means of ILP code." In this research a polynomial-time algorithm for TSPs is introduced. The proposed algorithm has been effectively used for the values of n of order 10 when running examples presented in Section 4 on a standard personal computer.

Section 2.1 considers an algorithm for the solution of the subset sum problem. Section 2.2 introduces a simple sub-tours elimination technique. Section 3 describes the proposed algorithm. Section 4 provides results of computer experiments. Section 5 discusses the analysis and concludes the paper.

2. Theoretical Background

2.1 The Subset Sum Problem

Let $a_1, a_2, \dots, a_l, l \in \boldsymbol{N}$, be arbitrary positive integers. The subset sum problem for any sum $L \in \boldsymbol{N}$ means to find all existing vectors $(s_1, s_2, \dots, s_l)^T$ with $s_i \in \{0,1\}, i = 1, \dots, l$, such that

$$a_1 s_1 + a_2 s_2 + \cdots + a_l s_l = L. \tag{2.1}$$

Voinov and Nikulin (1997) introduced an algorithm that using the corresponding generating function and the binomial theorem enumerates all nonnegative integer solutions of Equation (2.1). All 0-1 solutions to Equation (2.1) can be ascertained using the aforementioned algorithm with the generating function

$$\Psi_L(z) = (z^{a_1} + z^{a_2} + \cdots + z^{a_l})^L = \sum_{k=L\min(a_i)}^{L\max(a_i)} R_k(L,l),$$

where

$$R_k(L,l) = \sum_{s_l=0}^{\min\left(1,\left[\frac{L}{a_l}\right]\right)} \sum_{s_{l-1}=0}^{\min\left(1,\left[\frac{L-a_l s_l}{a_{l-1}}\right]\right)} \cdots \sum_{s_2=0}^{\min\left(1,\left[\frac{L-a_l s_l \cdots - a_3 s_3}{a_2}\right]\right)} \frac{L!}{(L-s_1-\cdots-s_l)!\, s_1! \cdots s_l!} \tag{2.2}$$

and $s_1 = \frac{L - a_l s_l - \cdots - a_2 s_2}{a_1}$ is necessarily either 0 or 1. Otherwise one concludes that there are no solutions of Equation (2.1). The notation $[a]$ denotes the greatest integer part of a. The right-hand side multiplier in (2.2) presents the total number of compositions (total number of partitions, taking into account the order of terms) that satisfy the above condition. If the value of that multiplier is set to 1, Equation (2.2) gives the number *N* of 0-1 solutions for Equation (2.1). The solutions, if they exist, are written explicitly as

$$\{a_1^{s_1}, a_2^{s_2}, \dots, a_l^{s_l}\}, \quad (2.3)$$

where $s_2, s_3, \dots, s_l$ is the set of 0-1 variables such that (2.2) is summed over, with $s_1 = \frac{L - a_l s_l - \cdots - a_2 s_2}{a_1} \in \{0,1\}$. The notation (2.3) means that in a particular partition (a solution of Equation (2.1)) there will be s_1 terms equal to a_1, s_2 terms of a_2, and so on.

The algorithm given by formulas (2.2) and (2.3) was realized as the R-function "get.subsetsum" of the R-package "nilde" (see Pya Arnqvist, Voinov, Makarov, and Voinov, 2021). Since the algorithm in (2.2) is defined by a sequence of nested 0-1 sums, its time complexity is $O(l^2)$.

Lambe (1977) derived a tight upper bound on the number $\mathcal{N}$ of nonnegative integer solutions to the equation

$$a_1 s_1 + a_2 s_2 + \cdots + a_l s_l = L \quad (2.4)$$

with $s_i \in Z_{\geq 0}$ as

$$\mathcal{N} \leq \binom{l-1+B}{l-1} \frac{1}{\prod_{i=1}^{l} a_i} = \frac{l(l+1)\cdots(l+B-1)}{B! \prod_{i=1}^{l} a_i}, \quad (2.5)$$

where

$$B = L + \frac{a_1 a_2}{f_2} - 1 + \sum_{i=3}^{l} \left[\frac{a_i f_{i-1}}{2 f_i}\right], \quad (2.6)$$

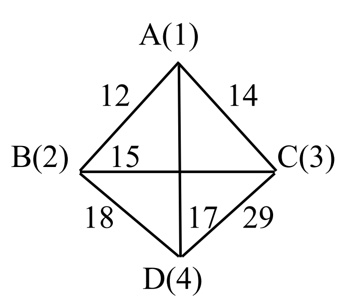
and f_i is the largest common factor of sets $\{a_1, \dots, a_i\}, i = 2, \dots, l$, with $f_l = 1$.

From (2.5) we see that the upper bound for $\mathcal{N}$ is polynomial in *l* of order *B*. This polynomial in *l* bound can be used for the *N* 0-1 solutions to Equation (2.1) because they form a subset of all $\mathcal{N}$ nonnegative integer solutions for (2.4).

2.2 Sub-Tours Elimination

The algorithm in (2.2) and (2.3) permits us to enumerate explicitly all feasible solutions. This makes it possible to create a simple polynomial-in-time procedure for sub-tours elimination. To illustrate this approach, consider Example 5 from Martin (2014). The graph of this symmetrical TSP is given in Figure 1.

Figure 1
The Graph of the TSP



The cost or distance matrix $C = (c_{ij})$ that corresponds to the above graph and the matrix of corresponding variables for Equation (2.1) are shown in Figure 2.

Figure 2

(a) Cost or Distance Matrix C; (b) Matrix of Variables in (2.1)

#\#	1	2	3	4
1		$c_{12}=12$	$c_{13}=14$	$c_{14}=17$
2	$c_{21}=12$		$c_{23}=15$	$c_{24}=18$
3	$c_{31}=14$	$c_{32}=15$		$c_{34}=29$
4	$c_{41}=17$	$c_{42}=18$	$c_{43}=29$	

(a)

#\#	1	2	3	4
1		s_4	s_7	s_{10}
2	s_1		s_8	s_{11}
3	s_2	s_5		s_{12}
4	s_3	s_6	s_9	

(b)

When searching for an optimal solution of any TSP the lower and upper bounds on the tour length L are of importance. The best lower bound Lb is obtained by solving the corresponding assignment problem (Laporte, 1992, p. 234). The upper bound Ub can be obtained "by means of a suitable heuristic" (Laporte, 1992, p. 238). For the above example $Lb = 63$. The "cheapest-insertion" algorithm of the R-package "TSP" gives $Ub = 64$. From this it follows that the optimal tour length for our instance is $L = 63$ or $L = 64$.

Let us check these values for optimality, using the ability to construct explicitly all 0-1 solutions of Equation (2.1) and the sub-tours elimination technique considered below.

For convenient usage of formulas in Sections 1 and 2.1, the variables $\delta_{ij}, i,j = 1,2,3,4$, are renamed as follows: $\delta_{21}=s_1$, $\delta_{31}=s_2$, $\delta_{41}=s_3$, $\delta_{12}=s_4$, $\delta_{32}=s_5$, $\delta_{42}=s_6$, $\delta_{13}=s_7$, $\delta_{23}=s_8$, $\delta_{43}=s_9$, $\delta_{14}=s_{10}$, $\delta_{24}=s_{11}$, $\delta_{34}=s_{12}$. Under these notations the length of tour L in (1.1) and constraints in (1.2) and (1.3) are written down explicitly as

$$c_{21}s_1+c_{31}s_2+c_{41}s_3+c_{12}s_4+c_{32}s_5+c_{42}s_6+c_{13}s_7+c_{23}s_8+c_{43}s_9+c_{14}s_{10}+c_{24}s_{11}+c_{34}s_{12}$$
$$=12s_1+14s_2+17s_3+12s_4+15s_5+18s_6+14s_7+15s_8+29s_9+17s_{10}+18s_{11}+29s_{12}=L \quad (2.7)$$

and

$$\begin{aligned} s_1+s_2+s_3&=1,\\ s_4+s_5+s_6&=1,\\ s_7+s_8+s_9&=1,\\ s_{10}+s_{11}+s_{12}&=1, \end{aligned} \quad (2.8)$$

$$\begin{aligned} s_4+s_7+s_{10}&=1,\\ s_1+s_8+s_{11}&=1,\\ s_2+s_5+s_{12}&=1,\\ s_3+s_6+s_9&=1 \end{aligned} \quad (2.9)$$

respectively. For $L = 63$ the R-command
get.subsetsum(a=c(12,14,17,12,15,18,14,15,29,17,18,29),M=12,n=63,problem="subsetsum01")
from the R-package "nilde" produces 14 0-1 solutions of the equation in (2.7), but no one of them satisfies degree constraints in (2.8) and (2.9). Thus, $L = 63$ cannot be the optimal solution.

Let now $L = 64$. For $L = 64$ the R-command
get.subsetsum(a=c(12,14,17,12,15,18,14,15,29,17,18,29),M=12,n=64,problem="subsetsum01")
from the R-package "nilde" produces 30 0-1 solutions of the equation in (2.7). Four of them that satisfy the degree constraints in (2.8) and (2.9) are:

$$s_2 + s_6 + s_7 + s_{11} = 4 \text{ or } c_{31} + c_{42} + c_{13} + c_{24} = 64, \tag{2.10}$$

$$s_3 + s_5 + s_7 + s_{11} = 4 \text{ or } c_{41} + c_{32} + c_{13} + c_{24} = 64, \tag{2.11}$$

$$s_2 + s_6 + s_8 + s_{10} = 4 \text{ or } c_{31} + c_{42} + c_{23} + c_{14} = 64, \tag{2.12}$$

$$s_3 + s_5 + s_8 + s_{10} = 4 \text{ or } c_{41} + c_{32} + c_{23} + c_{14} = 64. \tag{2.13}$$

To eliminate possible sub-tours, the following simple explicit procedure can be used. Consider first the solution in (2.10). The first summand c_{31} on the right-hand side of (2.10) means that a salesman has to pass a way from, say, city 3 to 1. The third summand c_{13} returns him back to city 3. Thus we have a sub-tour $c_{31} + c_{13}$ of size 2. It follows that there is no need to analyze the other summands and thus this solution is not a Hamiltonian circuit, and it has to be removed from consideration. Having rearranged the summands on the right-hand side of (2.11) as $c_{41} + c_{13} + c_{32} + c_{24} = 64$, one sees that this Hamiltonian circuit represents the optimal solution of the problem.

By analogy, the right-hand side of (2.12) which equals $c_{31} + c_{14} + c_{42} + c_{23} = 64$ is also a Hamiltonian circuit that represents the second optimal solution. Note that this solution presents the same circuit as in (2.11) but passed in the opposite direction. The solution in (2.13) contains a sub-tour $c_{41} + c_{14}$ and should be removed. From the above it follows that the TSP under consideration (as per the “cheapest-insertion” algorithm) has two optimal solutions (2.11) and (2.12) that can be presented as 1,3,2,4,1 or ACBDA and 1,4,2,3,1 or ADBCA. Note that actually we have 8 dependent on starting vertex optimal solutions: ACBDA, CBDAC, BDACB, DACBD, ADBCA, DBCAD, BCADB, and CADBC.

Remark. The sub-tours elimination procedure is expected to be polynomial-time because the number of solutions satisfying the degree constraints is less than or equal to $\mathcal{N}$ which in accordance with (2.5) is polynomial in $l = n(n-1)$.

3. A Description of the Algorithm

Step 1. (Initialization) Solve a corresponding assignment problem to obtain a lower bound on the value of the optimal TSP solution. Apply a heuristic to get an upper bound.
Step 2. (Sub problem solution) Given the lower bound, construct all N 0-1 solutions to a corresponding linear Diophantine equation.
Step 3. (Degree constraints check) Remove solutions that do not satisfy the degree constraints (1.2) and (1.3).
Step 4. (Sub-tours elimination) Remove solutions that contain sub-tours by applying the procedure explained in Section 2.2. If there is a solution or solutions that contain no sub-tours, it is the optimal solution or solutions: Stop. Otherwise, increase the lower bound by one and go to step 2. Repeat until the upper bound is reached.

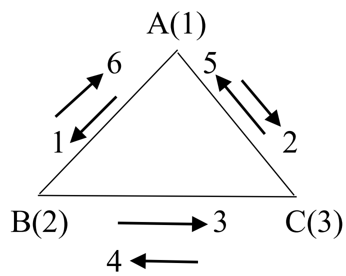
This algorithm was realized as the R-function tsp_solver() of the R-package “nilde” version 1.1-4 (Pya Arnqvist, Voinov, Makarov, & Voinov, 2021).

4. Computer Experiments

(a) As a further example consider the following 3-dimensional asymmetric TSP from Wikipedia (*en.wikipedia/wiki/Travelling_salesman_problem*)

Figure 3

The TSP Graph



with the following matrices:

Figure 4

Costs and Variables

#\#	1	2	3
1		$c_{12} = 1$	$c_{13} = 2$
2	$c_{21} = 6$		$c_{23} = 3$
3	$c_{31} = 5$	$c_{32} = 4$	

#\#	1	2	3
1		s_3	s_5
2	s_1		s_6
3	s_2	s_4	

The "cheapest-insertion" algorithms of the R-package "TSP" give the solution ABCA with L=9 that can be taken as Ub. The lower bound for this instance is $Lb = 8$.
The equation

$$6s_1 + 5s_2 + s_3 + 4s_4 + 2s_5 + 3s_6 = 8$$

possesses 4 0-1 solutions, but no one of them satisfies the degree constraints

$$\begin{aligned} s_1 + s_2 = 1, &\quad s_3 + s_5 = 1, \\ s_3 + s_4 = 1, &\quad s_1 + s_6 = 1, \\ s_5 + s_6 = 1, &\quad s_2 + s_4 = 1. \end{aligned} \tag{4.1}$$

The equation

$$6s_1 + 5s_2 + s_3 + 4s_4 + 2s_5 + 3s_6 = 9$$

possesses 5 0-1 solutions. One of them ($s_2 + s_3 + s_6 = 3$) that satisfies the degree constraints in (4.1) is $c_{31} + c_{12} + c_{23} = 9$ (ABCA). It has no sub-tours, and hence the solution 3,1,2,3 or ABCA is optimal. Note that Wikipedia gives as optimal the incorrect solution ACBA. It has to be noted also that "solving an asymmetric TSP graph can be somewhat complex" is an incorrect opinion of Wikipedia because our algorithm solves that graph easily.

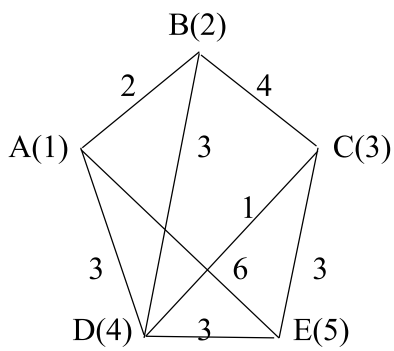
This example shows that the algorithm of Section 3 can also be used for asymmetrical TSPs.

(b) Goyal (2010) presented a greedy non-deterministic polynomial-in-time ($O(n^5)$) algorithm for solving TSPs and noted that it "halts with a minimum spanning tree of a graph instead of the

Hamiltonian Cycle in few cases." As an example of such an instance, Goyal (2010) used the disconnected graph in Figure 5.

Figure 5

The TSP Graph



Costs and variables for this graph are presented in Figure 6.

Figure 6

Costs and Variables

#\#	1	2	3	4	5
1		$c_{12} = 2$	-	$c_{14} = 3$	$c_{15} = 6$
2	$c_{21} = 2$		$c_{23} = 4$	$c_{24} = 3$	-
3	-	$c_{32} = 4$		$c_{34} = 1$	$c_{35} = 3$
4	$c_{41} = 3$	$c_{42} = 3$	$c_{43} = 1$		$c_{45} = 3$
5	$c_{51} = 6$	-	$c_{53} = 3$	$c_{54} = 3$	

#\#	1	2	3	4	5
1		s_4	-	s_{10}	s_{14}
2	s_1		s_7	s_{11}	-
3	-	s_5		s_{12}	s_{15}
4	s_2	s_6	s_8		s_{16}
5	s_3	-	s_9	s_{13}	

For this graph the lower bound is $Lb = 11$. The "cheapest-insertion" algorithms of the R-package "TSP" give the solution 1,4,5,3,2,1 or ADECBA with $L = 15$ respectively. So, the upper bound is $Ub = 15$. The equation

$$2s_1 + 3s_2 + 6s_3 + 2s_4 + 4s_5 + 3s_6 + 4s_7 + s_8 + 3s_9 + 3s_{10} + 3s_{11} + s_{12} + 3s_{13} + 6s_{14} + 3s_{15} + 3s_{16} = 15$$

possesses 1,392 0-1 solutions. $n_* = 46$ of them satisfy the degree constraints

$$\begin{aligned} s_1 + s_2 + s_3 = 1, &\quad s_4 + s_{10} + s_{14} = 1, \\ s_4 + s_5 + s_6 = 1, &\quad s_1 + s_7 + s_{15} = 1, \\ s_7 + s_8 + s_9 = 1, &\quad s_5 + s_{12} + s_{15} = 1, \\ s_{10} + s_{11} + s_{12} + s_{13} = 1, &\quad s_2 + s_6 + s_8 + s_{16} = 1, \\ s_{14} + s_{15} + s_{16} = 1, &\quad s_3 + s_9 + s_{13} = 1 \end{aligned} \tag{4.2}$$

and $n_{**} = 4$ satisfy also the connectivity constraints. These four Hamiltonian circuits are: $c_{41} + c_{12} + c_{23} + c_{54} + c_{35} = 15$ or 1,2,3,5,4,1 (ABCEDA), $c_{51} + c_{12} + c_{43} + c_{24} + c_{35} = 15$ or 1,2,4,3,5,1 (ABDCEA), $c_{21} + c_{42} + c_{53} + c_{34} + c_{15} = 15$ or 1,5,3,4,2,1 (AECDBA), and $c_{21} + c_{32} + c_{53} + c_{14} + c_{45} = 15$ or 1,4,5,3,2,1 (ADECBA). Since there are no other solutions satisfying all constraints for $L < 15$, these four are optimal. The same results are obtained by the following R-commands:

library(nilde); s=c(0,2,NA,3,6,2,0,4,3,NA,NA,4,0,1,3,3,3,1,0,3,6,NA,3,3,0); d<-matrix(s,5,5); g<-tsp_solver(d); g$tour.

This example shows how the proposed algorithm is used for TSPs with disconnected graphs.

(c) Consider Examples 1 and 4 used by Martin (2014) to compare the effectiveness of two heuristic algorithms, the "repetitive-nn" (RNNA) and the "cheapest-link" (CLA), versus the brute-force search. Concerning the 5-dimensional symmetric TSP of Example 1 the author concluded that the RNNA produces a better result (*L*=34), but it is still higher than the optimal *L*=32.

This example is described by the matrices

Figure 7
Costs and Variables

#\#	1	2	3	4	5
1		$c_{12} = 12$	$c_{13} = 10$	$c_{14} = 19$	$c_{15} = 8$
2	$c_{21} = 12$		$c_{23} = 3$	$c_{24} = 7$	$c_{25} = 2$
3	$c_{31} = 10$	$c_{32} = 3$		$c_{34} = 6$	$c_{35} = 20$
4	$c_{41} = 19$	$c_{42} = 7$	$c_{43} = 6$		$c_{45} = 4$
5	$c_{51} = 8$	$c_{52} = 2$	$c_{53} = 20$	$c_{54} = 4$	

#\#	1	2	3	4	5
1		s_5	s_9	s_{13}	s_{17}
2	s_1		s_{10}	s_{14}	s_{18}
3	s_2	s_6		s_{15}	s_{19}
4	s_3	s_7	s_{11}		s_{20}
5	s_4	s_8	s_{12}	s_{16}	

The costs of Figure 7 give $Lb = 32$. The "cheapest-insertion" algorithm gives $Ub = 33$. In this case the equation

$$12s_1 + 10s_2 + 19s_3 + 8s_4 + 12s_5 + 3s_6 + 7s_7 + 2s_8 + 10s_9 + 3s_{10} + 6s_{11} + 20s_{12} + 19s_{13} + \\ +7s_{14} + 6s_{15} + 4s_{16} + 8s_{17} + 2s_{18} + 20s_{19} + 4s_{20} = 32$$

has *N*=884 0-1 solutions. $n_* = 21$ of them satisfy the degree constraints

$$\begin{array}{ll} s_1 + s_2 + s_3 + s_4 = 1, & s_5 + s_9 + s_{13} + s_{17} = 1, \\ s_5 + s_6 + s_7 + s_8 = 1, & s_1 + s_{10} + s_{14} + s_{18} = 1, \\ s_9 + s_{10} + s_{11} + s_{12} = 1, & s_2 + s_6 + s_{15} + s_{19} = 1, \\ s_{13} + s_{14} + s_{15} + s_{16} = 1, & s_3 + s_7 + s_{11} + s_{20} = 1, \\ s_{17} + s_{18} + s_{19} + s_{20} = 1, & s_4 + s_8 + s_{12} + s_{16} = 1 \end{array} \qquad (4.3)$$

and only 2 solutions: $c_{31} + c_{42} + c_{23} + c_{54} + c_{15} = 32$ (3,1,5,4,2,3) and $c_{51} + c_{32} + c_{13} + c_{24} + c_{45} = 32$ (5,1,3,2,4,5) satisfy the connectivity constraints (*Step 4*) of Section 3. These two solutions represent the same Hamiltonian circuit but pass in opposite directions. Since there are no optimal solutions for $L < 32$, one may conclude that the above two Hamiltonian circuits are optimal.

Concerning the 6th-dimensional symmetric TSP of Example 4 with the cost matrix

Figure 8
Costs for the TSP

#\#	1	2	3	4	5	6
1		$c_{12} = 12$	$c_{13} = 29$	$c_{14} = 22$	$c_{15} = 13$	$c_{16} = 24$
2	$c_{21} = 12$		$c_{23} = 19$	$c_{24} = 3$	$c_{25} = 25$	$c_{26} = 6$
3	$c_{31} = 29$	$c_{32} = 19$		$c_{34} = 21$	$c_{35} = 23$	$c_{36} = 28$
4	$c_{41} = 22$	$c_{42} = 3$	$c_{43} = 21$		$c_{45} = 4$	$c_{46} = 5$
5	$c_{51} = 13$	$c_{52} = 25$	$c_{53} = 23$	$c_{54} = 4$		$c_{56} = 16$
6	$c_{61} = 24$	$c_{62} = 6$	$c_{63} = 28$	$c_{64} = 5$	$c_{65} = 16$	

Martin (2014) concluded that CLA produces a better result with $L = 83$ and that the optimal solution 1,3,6,2,4,5,1 is achieved at $L = 76$. Our approach gives *N*=18,822 0-1 solutions for the equation $12s_1 + 29s_2 + \cdots + 16s_{30} = 76$, $n_* = 147$ of them satisfy the degree constraints, and $n_{**} = 2$ satisfy the connectivity constraints. These two solutions are: $c_{51} + c_{32} + c_{13} + c_{64} + c_{45} + c_{26} = 76$ and $c_{31} + c_{62} + c_{23} + c_{54} + c_{15} + c_{46} = 76$. Since there are no optimal solutions for $L < 76$, one may conclude that these two (1,3,2,6,4,5,1 and 1,5,4,6,2,3,1) are optimal. Note that they represent the same Hamiltonian circuit passed in opposite directions.

(e) Consider the 7th-dimensional symmetric TSP with the cost matrix

Figure 9
Costs for the TSP

#\#	1	2	3	4	5	6	7
1		$c_{12} = 26$	$c_{13} = 31$	$c_{14} = 35$	$c_{15} = 33$	$c_{16} = 39$	$c_{17} = 41$
2	$c_{21} = 26$		$c_{23} = 29$	$c_{24} = 32$	$c_{25} = 38$	$c_{26} = 40$	$c_{27} = 60$
3	$c_{31} = 31$	$c_{32} = 29$		$c_{34} = 50$	$c_{35} = 42$	$c_{36} = 38$	$c_{37} = 45$
4	$c_{41} = 35$	$c_{42} = 32$	$c_{43} = 50$		$c_{45} = 60$	$c_{46} = 44$	$c_{47} = 42$
5	$c_{51} = 33$	$c_{52} = 38$	$c_{53} = 42$	$c_{54} = 60$		$c_{56} = 28$	$c_{57} = 45$
6	$c_{61} = 39$	$c_{62} = 40$	$c_{63} = 38$	$c_{64} = 44$	$c_{65} = 28$		$c_{67} = 30$
7	$c_{71} = 41$	$c_{72} = 60$	$c_{73} = 45$	$c_{74} = 42$	$c_{75} = 45$	$c_{76} = 30$	

For this instance, $Lb = 225$ and $Ub = 232$. This upper bound presents a solution of the problem obtained by the heuristic "cheapest-insertion" algorithm. The R-commands: library(nilde); s=c(26,31,35,…,42,45,30); d<-matrix(s,7,7); g<-tsp_solver(d); g$tour give two tours 3,1,5,6,7,4,2,3 5,1,3,2,4,7,6,5 of length *L*=225 which present the same Hamiltonian circuit passed in opposite directions. This instance confirms that no one known heuristic algorithm guarantees the optimality of a solution.

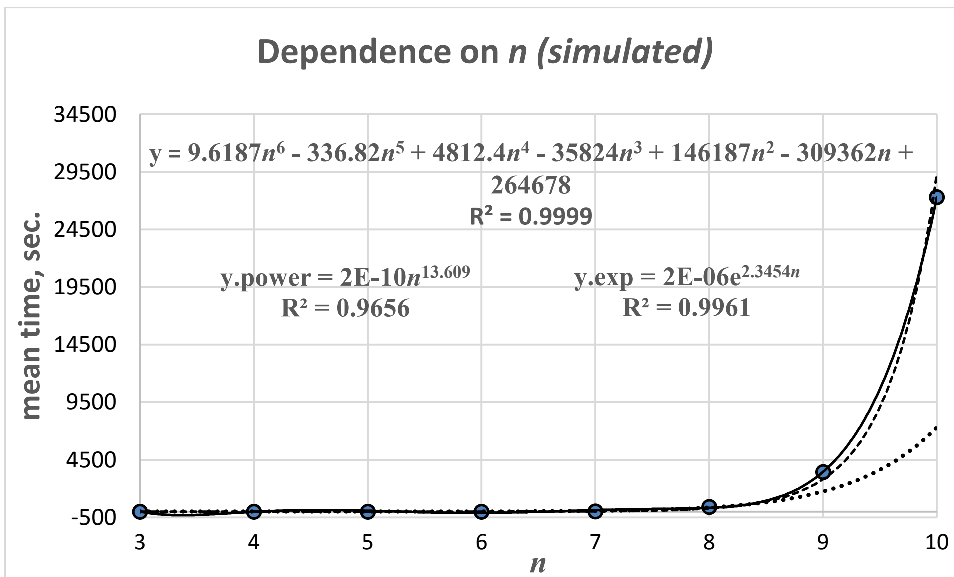
Note that the DFG approach requires checking $2^n - 2n - 2 = 112$ sub-tour elimination constraints against $n_{**} = 2$ for the proposed algorithm.

(e) *Assessing the time complexity.* Garey and Johnson (1978, p. 500) wrote that "the time complexity of an algorithm is expressed in terms of a single 'instance size' parameter which reflects the number of symbols that would be required to describe the instance in a 'reasonable' and 'concise' manner." For the proposed algorithm the single "instance size" parameter can be, e.g., $p = n + \max(c_{ij})$. It can be shown that for a few instances considered above, the time complexity is of the order of $O(p^3)$. For a systematical analysis of our algorithm's complexity, the following simulation experiments were designed:

(i) For every n from 3 to 10 inclusive we simulated at random 10 asymmetric TSPs with c_{ij} in the interval [1,100], solved problems using the function tsp_solver() from the R-package "nilde," and calculated mean computing times and their standard deviations of the mean (see the script used in the Appendix). Results (obtained on PC Intel® Xeon ® CPU E3-1280v5@3.70GHz, RAM 62.8Gb) are presented in Figure 10. The mean times' fitted curves were obtained by Microsoft Excel 2013.

Figure 10

Dependence of the Mean Computing Time on n



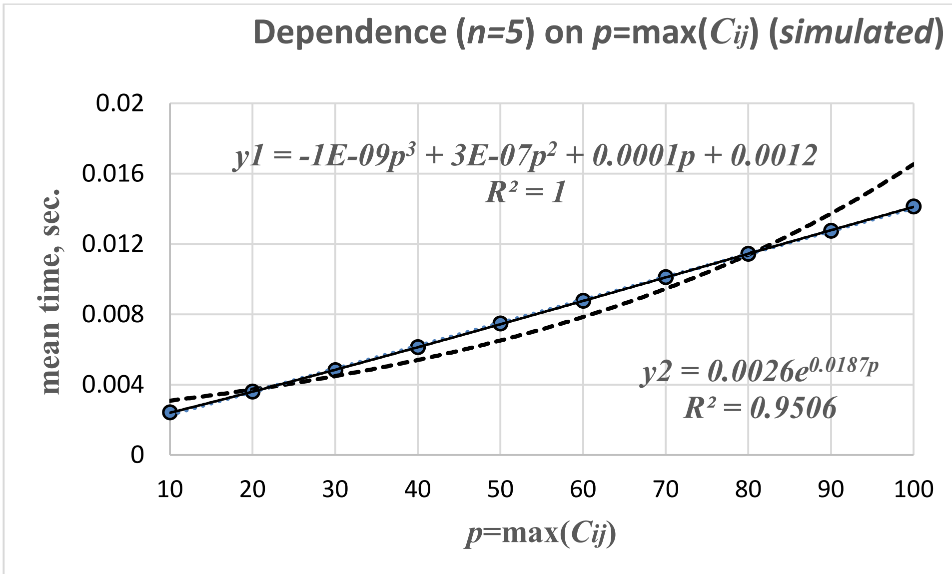
The solid line presents the polynomial fit, the dashed one is for the exponential, and the dotted line presents the power fitted line.

From Figure 10 one sees that for the polynomial fit the R^2 is higher than that for the power and exponential ones. In accordance with the formula for (y), the total contribution of terms $4812.4n^4, 146187n^2$, and constant = 264678 is about 90% of the total contribution of all positive values including $9.6187n^6$.

The above dependence of times on n is equivalent to that on $n + \max(c_{ij})$, because $\max(c_{ij}) \leq 100$ for all n = 3-10, and, hence, 100 can be taken as $\max(c_{ij})$.

(ii) For n = 5 and every $\max\left(c_{ij}\right) = 10, 20, \ldots, 100$ we simulated at random 2,000 asymmetric TSPs with c_{ij} in the interval [1,50] using the function tsp_solver() and calculated mean computing times and their standard deviations of the mean. Results (obtained on PC Intel® Core™ i7-2600 CPU@3.40GHz, RAM 6Gb) are presented in Figure 11.

Figure 11
Polynomial and Exponential Fits



The solid line y1 is for the polynomial fit, and the dashed y2 is for the exponential fit.

From Figure 11 one sees that for a polynomial fit the R^2 is larger than that for the exponential one. The above dependence of times on $\max(c_{ij})$ is equivalent to that on $n + \max(c_{ij})$, because n is the same for all 10 values of $\max(c_{ij})$.

The results of simulation show that mean computing times, being a polynomial function of the single parameter $p = n + \max(c_{ij})$, are in favor of belonging TSPs to the class P.

5. A Discussion and Conclusions

This research answers "yes" to the following routing decision problem (Garey & Johnson, 1979, p. 211):
[ND22] TRAVELING SALESMAN
INSTANCE: Set *C* of *n* cities, distances $d(c_i, c_j) \in Z^+$ for each pair of cities $c_i, c_j \in C$, positive integer *B*.
QUESTION: Is there a tour of *C* having length *L* or less, i.e., a permutation $< c_{\pi(1)}, c_{\pi(2)}, \dots, c_{\pi(n)} >$ of *C* such that $\left(\sum_{i=1}^{n-1} d(c_{\pi(i)}, c_{\pi(i+1)})\right) + d\left(c_{\pi(n)}, c_{\pi(1)}\right) \leq B$?

Our "yes" is confirmed by presenting a corresponding polynomial-time algorithm for solving TSPs, by numerous published and simulated graphs of Section 4. There was no one answer "no" for more than 20,000 simulated at random TSPs. Thus our results disprove the commonly accepted opinion that TSP is NP-complete.

In 1971 Stephen Cook showed that "if we have a polynomial time reduction from one problem to another, this ensures that any polynomial time algorithm for the second problem can be converted into a corresponding polynomial time algorithm for the first problem" (see Garey and Johnson, 1979, p. 13). Since there is a transformation from the Hamiltonian circuit (HC) problem to TSP (ibid., 211.), from the above result of Cook and our arguments in favor of polynomial-time complexity of TSPs, it follows that the HC problem also belongs to the class P. The same logic shows that importantly for the graph theory problems: [GT31] MINIMUM K-CONNECTED SUBGRAPHS and [GT34] HAMILTONIAN COMPLETION (ibid., 198) belong to the class P.

To summarize, the following strengths of the proposed algorithm can be emphasized: (a) it is polynomial in time with the complexity of $O(n^{<6})$; (b) it enumerates all existing exact optimal solutions for symmetrical and asymmetrical TSPs with both connected and disconnected graphs; (c) it uses new simple polynomial-time constraints for sub-tours elimination.

The main weakness of the algorithm is its rather high PC computing time for TSPs of size $n > 10$. This is explained by a high processor time needed for constructing all nonnegative 0-1 solutions of a linear Diophantine equation. One hopes that future research will develop much faster algorithms for enumerating those solutions. Since the computing time does not increase exponentially, the use of contemporary supercomputers permits one to solve TSPs of size $n > 10$ in a reasonable time.

From all the above we may conclude that TSPs being solvable in polynomial time are not NP-hard. This is an empirical argument in favor of the fundamental equality P=NP. Analogous arguments can be found in Voinov and Rahmanov (2020).

In 1964 Vassilly Voinov graduated from Tomsk State University in the former USSR. In 1989 he defended a doctoral thesis in physics and mathematical statistics in the Joint Institute for Nuclear Research (Dubna, Moscow region). He is a leading author of six books in mathematical statistics and experimental nuclear physics. Three of them have been published by Kluwer and Academic Publishers. Since 1998 he has been teaching 12 courses related to mathematical statistics at KIMEP University for undergraduate and graduate students. He is a member of the American Statistical Association and the American Mathematical Society. He has published about 110 research papers. He has also one invention, and three published contributions in mathematical software.

In 1999 Natalya Pya Arnqvist graduated from Al-Farabi Kazakh State National University (Almaty, Kazakhstan) with an MSc degree in Mathematics, Theory of Probability and Mathematical Statistics. In 2005 she obtained a Candidate of Physical and Mathematical Sciences Degree from the Institute of Mathematics of the Ministry of Education and Sciences of the Republic of Kazakhstan. In 2010 Natalya obtained a PhD in Statistics at the Department of Mathematical Sciences, University of Bath, UK. She has published 22 articles and five book chapters. She is an author and creator of four extensions to the R statistical programming language.

6. Summary

English: This paper proposes an exact polynomial-in-time algorithm for the optimal solution of traveling salesman problems.

Russian: В статье предложен точный полиномиальный алгоритм для оптимального решения задач коммивояжера.

Kazakh: Мақалада сатушының есептерін оңтайлы шешу үшін нақты полиномиялық алгоритм ұсынылған.

7. References

Dantzig, G., Fulkerson, D., & Johnson, S. (1954). Solution of a large-scale traveling-salesman problem. *Operations Research, 2*, 393-410. Retrieved from https://doi.org/10.1287/opre.2.4.393

Garey, M., & Johnson, D. (1978). “Strong” NP-completeness results: Motivation, examples, and implications. *Journal of the ACM, 25(3)*, 499-508. Retrieved from https://doi.org/10.1145/322077.322090

Garey, M., & Johnson, D. (1979). *Computers and intractability: A guide to the theory of NP-completeness.* New York: W. H. Freeman & Co. Retrieved from https://doi.org/10.1137/1024022

Goyal, S. (2010). *A survey on Traveling Salesman Problem*. Retrieved from http://www.semanticscholar.org, corpus ID: 14205511.

Lambe, T. (1977). Upper bound on the number of nonnegative integer solutions to a linear equation. *SIAM Journal on Applied Mathematics, 32(1)*, 215-219. Retrieved from https://doi.org/10.1137/0132018

Laporte, G. (1992). The traveling salesman problem: An overview of exact and approximate algorithms. *European Journal of Operational Research, 59*, 231-247. Retrieved from https://doi.org/10.1016/0377-2217(92)90138-Y

Martin, J. (2014). *Examples of Traveling Salesman Problems*. Retrieved from http://www.jlmartin.faculty.ku.edu/~jlmartin/courses/math105-F14/chapter6-part6.pdf

Pataki, G. (2003). Teaching integer programming formulations using the Traveling Salesman Problem. *SIAM Review, 45*, 116-123. Retrieved from https://doi.org/10.1137/S00361445023685

Pya Arnqvist, N., Voinov, V., Makarov, R., & Voinov, Y. (2021). “nilde”: Non-negative integer solutions of linear Diophantine equations with applications. R-package version 1.1-4. Retrieved from https://CRAN, R-project.org/package-nilde

Sathya, N., & Muthukumaravel, A. (2015). A review of the optimization algorithms on Traveling Salesman Problem. *Indian Journal of Science and Technology, 8(29)*, 1-4. Retrieved from https://doi.org/ 10.17485/ijst/2015/v8il/84652

Voinov, V., & Nikulin, M. (1997). On a subset sum algorithm and its probabilistic and other applications. In N. Balakrishnan (Ed.), *Advances in combinatorial methods and applications to probability and statistics* (153-163). Boston: Birkhäuser.

Voinov, V., & Rahmanov, M. (2020). 2-, 3-, and 4-partition problems and their relation to the equality P=NP, *Central Asia Business Journal, 11(2)*, 34-46.

Wikipedia. Traveling Salesman Problem. (n.d.). Retrieved from en.wikipedia/wiki/Travelling_salesman_problem. Accessed March 8, 2021.

Appendix
R Code

```
library(nilde)
cpu.tsp <- numeric(0)
for (j in 1:10){
 set.seed(j)
 d <- matrix(sample(1:100,25,replace=TRUE),5,5)
 cpu.tsp[j] <- system.time(g <- tsp_solver(d))[1]
}
m<-mean(cpu.tsp)
m
v<-var(cpu.tsp)
st.dev.mean<-v^(1/2)/10^(1/2)
st.dev.mean
```

John Maynard Keynes, *The General Theory of Employment, Interest, and Money*: A Review-Essay

Chingiz Mussin,
KIMEP University, Kazakhstan
chingiz.mussin@kimep.kz

Countries of Central Asia have only recently switched to a free-market system, yet the implications have already been felt by the economies of the region. Countries in this part of the world have experienced inflation, recessions, economic growth, and crises. To understand their possible impacts, consider the leading macroeconomic book of the 20th century, *The General Theory of Employment. Interest, and Money*, by John Maynard Keynes.

The work was published in the era of the Great Depression, when a laissez-faire approach to the economy, which had become popular during the Industrial Revolution, was still widespread (Barro & Gordon, 1983). This doctrine holds that the government should not interfere in the markets because these create more well-being when left untouched. In the classical theory of economics, a recession returns the economy to equilibrium because market forces decrease wage costs, and the excess capital is invested, resurrecting the economy (Blanchard, 2011). The "invisible hand" of self-interest guides the market to the outcome that benefits society, through economic growth (Barro & Gordon, 1983). The term "ínvisible hand" was introduced in 1776 by Adam Smith (1976) in his foundational work of economics, *An Inquiry into the Nature and Causes of the Wealth of Nations*.

Contrary to classical theory, Keynes argued that the economy does not operate at full employment. He discarded Say's Law that supply creates demand and held instead that the government should intervene in markets to stimulate demand in recessions (Gul, Chanudhry, & Faridi, 2014). The fall of prices in a recession fails to stimulate production, because wages are sticky-downward. Firms cannot afford to step up production when revenues fall and costs don't. Furthermore, enterprises are not willing to make capital investments because they are cautious of the decrease in consumer demand.

The English economist argued that a national economy may long remain in the doldrums, with high unemployment. We can see why by analyzing the aggregate output. It has four elements: Consumption, investment, government purchases, and net exports. Because consumption and investment fall during a recession, only the government is left with the ability to influence the aggregate demand. Keynes believed that the government should moderate the business cycles to keep the economy from entering a prolonged recession, and also moderate it during booms to avoid overheating the economy (Jahan, Mahmud, & Papageorgiou, n.d.).

Three principles guide Keynesian economic theory. First, aggregate demand is influenced by a lot of factors, both private and public. A mixed economy led by private enterprise and supported by the government is ideal. Second, wages and prices are not quick to react to changes in the economy, so in the short term there are either shortages or surpluses. Third, Keynesians believe that business cycles have the greatest impact on employment and output because the prices are sticky.

Keynes laid out several ideas for how the government can avoid a lengthened decrease in aggregate demand. One idea contradicted the conventional wisdom that the government should not always strive to balance the budget. Instead, it should offset current market conditions. If the economy is in a recession, the government should engage in deficit spending to increase investment and spending. In addition, Keynes accepted the idea of a multiplier effect, i.e., that a round of spending will stimulate more rounds, increasing output. Finally, monetary and fiscal policy should work together to avoid reductions in aggregate demand. For example, the central bank may reduce interest rates to stimulate investment while the fiscal authorities do not increase taxes to counteract a budget deficit.

In short, Keynesian economics looks to the government to raise aggregate demand when the private sector falls short of full employment.

Keynes had a major impact on economic thinking as well as on fiscal and monetary policies worldwide. During World War II, the United States government stepped up spending and subsidies, reducing the unemployment rate to virtually zero and increasing gross domestic product (GDP) by 15% year over year during the first three years of the 1940s (Ohanian, 1997). The same principles can be seen in the economic stimulus packages delivered by governments around the world in response to the financial crisis of 2008. In the Economic Stimulus Act (2008), the US paid residents, rebated taxes, and allowed federal securitizers to purchase more expensive mortgages, all to increase aggregate demand.

Neither did Central Asia abstain from government support. Kazakhstan's government was the first to act. Beginning in 2007, it financed banks and the real estate sector to save construction jobs (Overseas Development Institute, 2009). In 2009, it cut taxes on non-extractive enterprises (United States Department of State, n.d.). Meanwhile, Uzbekistan cut taxes to support small and medium-sized enterprises, and reduced the personal income tax. In the prior year, its fiscal stimulus amounted to around 4% of GDP (EBRD, 2009), including credit for exporters (World Bank, n. d.). Tajikistan's stimulus, 3.4% of GDP, targeted the underprivileged (Asian Development Bank, 2010). Kyrgyzstan obtained international support for its pay hikes to public workers and for pension increases (World Bank Group, 2018).

Turkmenistan claimed that its economy grew 6.1% in 2009 despite the fall in demand for its hydrocarbon exports. To return to pre-crisis rates of GDP growth (11.4% in 2006 and 11.6% in 2007), the government spent more than before on education and social security as well as doubled capital spending (International Monetary Fund, 2010).

Central Asia has also taken a Keynesian tack to the COVID-19 pandemic. Kazakhstan arranged cheap loans to small and medium-sized enterprises, cut taxes, and paid residents directly in a stimulus amounting to 5.7% of GDP (World Bank Group, 2020). Uzbekistan approved interest-free loans to businesses and abandoned excise tax increases (KPMG, n.d., a). Tajikistan's response included the distribution of loans to companies that produce or supply food and medical goods, in addition to lowering the interest rates and the reserve requirements for banks (KPMG, n.d., b). The central authorities of Turkmenistan revised the budget to assist the organizations harmed by the restrictive measures (KPMG, n.d., d). The government of Kyrgyzstan pushed back deadlines for submission of taxes, introduced a moratorium on audits by the authorities, and collected foreign aid that supported firms (KPMG, n.d., c).

In Central Asia, John Maynard Keynes is alive and well.

Chingiz Mussin is a third-year bachelor student majoring in Corporate Finance and Investments at the Bang College of Business of KIMEP University. His research interests include public financial policy as well as organization of efficient business practices in Central Asia.

References

Asian Development Bank. (2010). Tajikistan: Crisis recovery support program. Retrieved from https://www.adb.org/sites/default/files/project-document/62247/43098-01-taj-pcr.pdf

Barro, R. J., & Gordon, D. (1983). Rules, discretion, and reputation in a model of monetary policy. *Journal of Monetary Economics, 12(1)*, 101–21. Retrieved from https://doi.org/10.1016/0304-3932(83)90052-1

Blanchard, O. (2011). *Macroeconomics*. Hoboken, New Jersey: Prentice Hall.

European Bank of Reconstruction and Development. (2009). Uzbekistan – transition assessment. Retrieved from https://www.ebrd.com/downloads/research/economics/uzbekistan.pdf

Gul, E., Chanudhry, I.S., & Faridi, M.Z. (2014). The classical-Keynesian paradigm: Policy debate in contemporary era. Retrieved from https://mpra.ub.uni-muenchen.de/53920/2/MPRA_paper_53920.pdf

International Monetary Fund. (2010, November 5). IMF executive board concludes 2010 Article IV consultation with Turkmenistan. Retrieved from https://www.imf.org/en/News/Articles/2015/09/28/04/53/pn10146

Jahan, S., Mahmud A.S., & Papageorgiou, C. (n.d.). What is Keynesian economics? International Monetary Fund. Retrieved from https://www.imf.org/external/pubs/ft/fandd/basics/pdf/jahan_keynes.pdf

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money.* London, England: Macmillan.

KPMG. (n.d., a). *Uzbekistan: Tax developments in response to COVID-19.* Retrieved from https://home.kpmg/xx/en/home/insights/2020/04/uzbekistan-tax-developments-in-response-to-covid-19.html

KPMG. (n.d., b). *Tajikistan: Government and institution measures in response to COVID-19.* Retrieved from https://home.kpmg/xx/en/home/insights/2020/04/tajikistan-government-And-institution-measures-in-response-to-covid.html

KPMG. (n.d., c). *Kyrgyzstan: Government and institution measures in response to COVID-19.* Retrieved from https://home.kpmg/xx/en/home/insights/2020/04/kyrgyzstan-Government-and-institution-measures-in-response-to-covid.html

KPMG. (n.d., d). *Turkmenistan: Government and institution measures in response to COVID-19.* Retrieved from https://home.kpmg/xx/en/home/insights/2020/04/turkmenistan-government-and-institution-measures-in-response-to-covid.html

Ohanian, L. E., (1997). The macroeconomic effects of war finance in the United States: World

Mussin: Keynes's *General theory*

War II and the Korean War. *The American Economic Review, 87(1)*, 23-40.

Overseas Development Institute. (2009). *Impact of the economic crisis and food and fuel price volatility on children and women in Kazakhstan*.
Retrieved from https://cdn.odi.org/media/documents/5827.pdf

Smith, A. (1976). *An inquiry into the nature and causes of the wealth of nations*. Chicago: University of Chicago Press.

United States Congress. Economic Stimulus Act of 2008. (2008). Pub. L. No. 110–185, 122 STAT. 613. Retrieved from https://www.congress.gov/bill/110th-congress/house-bill/5140

United States Department of State, Bureau of Economic and Business Affairs. (n.d.). *2017 investment climate statements: Kazakhstan.* Retrieved from https://www.state.gov/reports/2017-investment-climate-statements/kazakhstan/

World Bank. (n.d.). *World Trade Indicators 2009/10: Uzbekistan trade brief.* Retrieved from http://documents1.worldbank.org/curated/en/213921468172484572/pdf/72794020090Uzb0Box0371958B00PUBLIC0.pdf

World Bank Group. (2018). *Kyrgyz Republic: From vulnerability to prosperity*. World Bank Publications.
Retrieved from http://documents1.worldbank.org/curated/en/516141537548690118/pdf/Kyrgyz-Republic-SCD-English-Final-August-31-2018-09182018.pdf

World Bank Group. (2020). *Navigating the crisis: Kazakhstan economic update.* Retrieved from http://documents1.worldbank.org/curated/en/155811595364689964/pdf/Kazakhstan-Economic-Update-Navigating-the-Crisis.pdf

Board of Editors